BC Calculus Review   Name _____

Review of Derivatives, Integrals, and Applications. Calculators should be used for arithmetic operations only.

Write out the derivative with respect to x of each of the following.

$y = \sin x$	$y = \cos x$	$y = \tan x$	$y = e^x$
$y = \csc x$	$y = \sec x$	$y = \cot x$	
$y = \sin^{-1} x$	$y = \tan^{-1} x$	$y = \sec^{-1} x$	
$y = \ln x$	$y = \log_a x$	$y = a^x$	

I. Find the derivative of the following. *Write out solution neatly on your own paper!*

1. $y = (x+2)^8 (x+3)^6$

2. $y = \sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}}$

3. $y = \dfrac{x}{\sqrt{9 - 4x}}$

4. $y = \dfrac{e^x}{1 + x^2}$

5. $y = \sin(\cos x)$

6. $y = \sin^{-1} e^x$

7. $y = x e^{\frac{1}{x}}$

8. $y = x^r e^{sx}$ (r and s are constants)

9. $y = \tan\sqrt{1-x}$

10. $y = \dfrac{1}{\sin(x - \sin x)}$

11. $y = \ln(\csc 5x)$

12. $y = \ln(x^2 e^x)$

13. $y = 5^{x \tan x}$

14. $x^2 y^3 + 3y^2 = x - 4y$

15. $y = \arctan(\arcsin \sqrt{x})$

16. Find f' in terms of g and g'. $f(x) = [g(x)]^2$

17. $\displaystyle \int_1^{x^2} \sqrt{1 + t^3}\, dt$

II. Find the definite or indefinite integral as indicated. *Write out solution neatly on your own paper!*

18. $\displaystyle \int_0^1 (1 - x^9)\, dx$

19. $\displaystyle \int (1 - x)^9\, dx$

20. $\displaystyle \int \dfrac{x}{x^2 + 1}\, dx$

21. $\displaystyle \int \dfrac{1}{x^2 + 1}\, dx$

22. $\displaystyle \int \dfrac{x^2 - x + 1}{\sqrt[3]{x^2}}\, dx$

23. $\displaystyle \int x^3 \ln x\, dx$

24. $\displaystyle \int \dfrac{1}{2 - 3x}\, dx$

25. $\displaystyle \int \sin x \cos(\cos x)\, dx$

26. $\displaystyle \int x^2 e^{-3x}\, dx$

27. $\displaystyle \int e^x \cos x\, dx$

28. $\displaystyle \int \dfrac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

29. $\displaystyle \int \dfrac{x}{9 + x^4}\, dx$

30. $\displaystyle \int \dfrac{1 - 3y}{\sqrt{2y - 3y^2}}\, dy$

31. $\displaystyle \int \cos 3x\, dx =$

32. $\displaystyle \int t \cos(2t)^2\, dt$

33. $\displaystyle \int \dfrac{\cos x\, dx}{\sqrt{1 + \sin x}}$

34. $\displaystyle \int \sec \dfrac{t}{2}\, dt$

35. $\displaystyle \int \dfrac{\ln v}{v}\, dv$

36. $\int lnx\, dx$ 　　　　　　　37. $\int \dfrac{y-1}{y+1}\, dy$ 　　　　　　　38. $\int \dfrac{5x-4}{2x^2+x-1}\, dx$

III. Applications

39. The angle of elevation of the sun is decreasing at a rate of .25 rad/hr. How fast is the

shadow cast by a 400 ft building increasing when the angle of elevation is $\dfrac{\pi}{6}$?

40. A liquid form of penicillin manufactured by a pharmaceutical firm is sold in bulk at a price of $200 per

unit. If the total production cost (in dollars) for x units is $C(x)=500,000+80x+0.003x^2$, and knowing

the Profit = Revenue-Cost, how many units of penicillin must be manufactured and sold to maximize profit?

41. A paper cup has the shape of a cone with height of 10 cm and radius 3 cm (at the top). If water is poured

into the cup at a rate of 2 cubic cm/s, how fast is the water level rising when the water is 5 cm deep?

42. Find the solution to the differential equations.

　　a. $\dfrac{dy}{dx}=y^2+1$ 　　　$y(0)=1$ 　　　　　　b. $\dfrac{dy}{dx}=\dfrac{1+x}{xy}$ 　　　$y(1)=-4$

　　c. $x+2y\sqrt{x^2+1}\,\dfrac{dy}{dx}=0$ 　　$y(0)=1$

43. The rate at which sewage enters a treatment tank is given by $E(t)=850+715\cos\left(\dfrac{\pi t^2}{9}\right)$ gallons per

hour for 0≤t≤4 hours. How many gallons of sewage are in the tank after 4 hours if 250 gallons are in there at

time t = 0?

44. The rate of change of bacteria growth is proportional to the amount present. If the count was 400 after 2

hours and 25,600 after 6 hours. (y = amount present, $\dfrac{dy}{dx}$ is the rate of growth, $\dfrac{dy}{dx}=ky$ is the rate of growth

is proportional to the amount present where k is the constant of proportionality.)

　　a. What was the initial population of the culture?
　　b. Find an expression for the population after t hours.
　　c. In what period of time does the population double?
　　d. When will the population reach 100,000?