Harrison High School
AB Calculus AP Prerequisite Packet

PROJECTED Optional help session for prerequisite material to be held at Harrison High School in Room 409 from9 am until noon on a morning the week before school starts. Please email me to confirm as this approaches--I have not yet been given the

To: AP Calculus AB Students and Parents
From: Michelle Bowman and Kim Watson, AP Calculus Instructors

The AP Course: AP Calculus AB is a college level course covering material traditionally taught in the first semester of college calculus. The course is taught in one semester consisting of 90 -minute classes. BC Calculus, corresponding to Calculus II at most universities, is taught during the second semester. Students are encouraged to take both semesters of AP Calculus.

The Prerequisite Packet: Students need a strong foundation to be ready for the rigorous work required throughout the term. Completing the prerequisite packet should prepare you for the material to be taught in the course. This packet consists of material studied during Algebra II and Precalculus. Students should anticipate working approximately 4 hours to complete it properly.
The packet will be collected on the first day of class for a 20 point completion grade. I will hand packets and your 20 point accuracy grade will be determined on neatness, completeness of solutions, and accuracy. In preparation for the AP test students need to begin showing all work with logical steps. Do not list only an answer. Work neatly and in an organized fashion.

Calculators: Students enrolled in AP Calculus AB will be using a graphing calculator throughout the course. A graphing calculator is required on the AP test. We will be using a TI 89 in class; we recommend a TI-89, but a TI 83 or TI 84 can also be used.

Assistance with the packet: There is a lot of help on the internet. There will also be an optional help session as indicated at the top of this page. This is a help session to work on problems you are having difficulty with, not a session in which to do your packet. Please complete as much of the packet as possible before attending the session. Students who will be on vacation during the help session and who have questions should contact me prior to this help session to determine whether an alternate time is available the week prior to school beginning. To obtain help please contact me by email at ***michelle.bowman@cobbk12.org***. You can also check my blog at the Harrison High School website for additional information. If you lose this packet another copy can be downloaded from the blog.

General Information: Students will have the opportunity to continue studying BC topics second semester through the BC Calculus course. The AB course is a prerequisite for BC. Students taking only the AB course and making a 3, 4, or 5 on the AB - AP test could receive 5 hours of credit for Calculus 1. Students taking both the AB and BC courses and passing the BC – AP test could earn up to 10 hours of college credit for Calculus 1 and Calculus 2. Be sure to check with your school of choice to determine their AP credit policy. Your success in this AP program will depend on the effort you put into the course. The work you do in the AB course is for you.

Since the AP test is offered only in May, students taking only the AB course should study for the test independently and attend any after school review sessions or practice test sessions that are offered. It is the student's responsibility to find out when these sessions will be held.

I anticipate a motivating and challenging year. Calculus is a stimulating and exciting field of mathematics and we look forward to sharing our excitement with you. I will be there to help and support you.

If you are downloading this packet and have not provided me with the following information, please email the information to Michelle.Bowman@cobbk12.org.

Name: _____

Address: _____

Phone: _____

Email: _____

Emailing this information acknowledges receipt of the packet and an understanding that completion of this prerequisite packet is a requirement for the AB Calculus course. The packet is due for a completion grade on the first day of class. We will work on the packet through the third day of class and then the packet will be due on the fourth day for an accuracy grade. We will have a quiz on the material after the packet is officially turned in on the fourth day.

Formulas and Identities

Trig Identities:

Reciprocal Identities:

$$\csc A = \frac{1}{\sin A} \qquad \sec A = \frac{1}{\cos A} \qquad \cot A = \frac{1}{\tan A}$$

Quotient Identities: $\tan A = \frac{\sin A}{\cos A}$ $\cot A = \frac{\cos A}{\sin A}$

Pythagorean Identities:

$\sin^2 A + \cos^2 A = 1$ $\tan^2 A + 1 = \sec^2 A$ $1 + \cot^2 A = \csc^2 A$

Double Angle Identities:

$\sin(2A) = 2\sin A \cos A$

$\cos(2A) = \cos^2 A - \sin^2 A$ $\cos(2A) = 2\cos^2 A - 1$ $\cos(2A) = 1 - 2\sin^2 A$

Polar Formulas:

$x^2 + y^2 = r^2$ $x = r\cos\theta$ $y = r\sin\theta$ $\tan^{-1}\frac{y}{x} = \theta \quad x > 0, \quad \tan^{-1}\frac{y}{x} = \theta + \pi \quad x < 0$

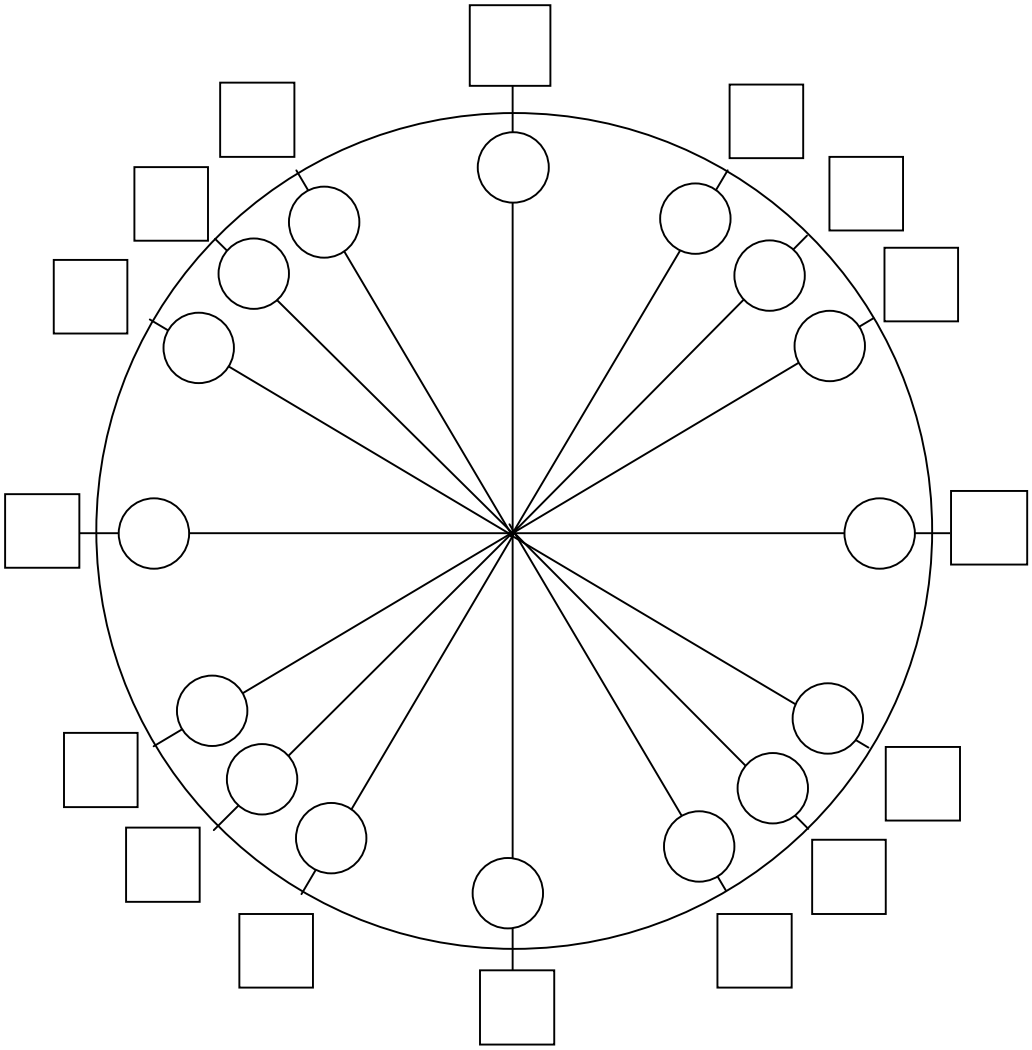
Geometric Formulas:

Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$ Area of a triangle: $A = \frac{1}{2}bh$

Area of an equilateral triangle: $A = \frac{\sqrt{3}}{4}s^2$

Area if a circle: $A = \pi r^2$ Circumference of a circle: $C = 2\pi r$ or $C = d\pi$

Unit Circle – Degrees and Radians



Place degree measures in the circles.

Place radian measure in the squares.

Place $(\cos\theta, \sin\theta)$ in parenthesis outside the square.

Place $\tan\theta$ outside the parenthesis.

$\tan\theta =$ _____

$\cot\theta =$ _____

$\csc\theta =$ _____

$\sec\theta =$ _____

SKILLS NEEDED FOR CALCULUS

I. Algebra:

*A. Exponents (operations with integer, fractional, and negative exponents)
*B. Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)
C. Rationalizing (numerator and denominator)
*D. Simplifying rational expressions
*E. Solving algebraic equations and inequalities (linear, quadratic, rational, radical, and absolute value equations)
F. Simultaneous equations

II. Graphing and Functions

*A. Lines (intercepts, slopes, write equations using point-slope and slope intercept, parallel, perpendicular, distance and midpoint formulas)
*B. Functions (definition, notation, domain, range, inverse, composition)
*C. Basic shapes and transformations of the following functions (absolute value, rational, root, higher order curves, log, ln, exponential, trigonometric. piece-wise, inverse functions)

III. Geometry

A. Pythagorean Theorem
B. Area Formulas (Circle, polygons, surface area of solids)
C. Volume formulas
D. Similar Triangles

*** IV. Logarithmic and Exponential Functions***

*A. Simplify Expressions (Use laws of logarithms and exponents)
*B. Solve exponential and logarithmic equations (include ln as well as log)
*C. Sketch graphs
*D. Inverses

*** V. Trigonometry***

**A. Unit Circle (definition of functions, angles in radians and degrees)
B. Use of Pythagorean Identities and formulas to simplify expressions and prove identities
*C. Solve equations
*D. Inverse Trigonometric functions
E. Right triangle trigonometry
*F. Graphs

VI. Limits

A. Concept of a limit
B. Find limits as x approaches a number and as x approaches ∞

* A solid working foundation in these areas is very important.

Calculus Prerequisite Problems

Work the following problems on your own paper. Show all necessary work.

I. Algebra

A. Exponents: 1) $\dfrac{\left(8x^3yz\right)^{1/3}(2x)^3}{4x^{1/3}\left(yz^{2/3}\right)^{-1}}$

B. Factor Completely:

2) $9x^2 + 3x - 3xy - y$ (use grouping) 3) $64x^6 - 1$ *Hint: Factor as difference of squares first, then as the sum and difference of cubes*

4) $42x^4 + 35x^2 - 28$ 5) $15x^{5/2} - 2x^{3/2} - 24x^{1/2}$ Hint: Factor GCF $x^{1/2}$ first.

6) $x^{-1} - 3x^{-2} + 2x^{-3}$ Hint: Factor out GCF x^{-3}

C. Rationalize denominator / numerator:

7) $\dfrac{3-x}{1-\sqrt{x-2}}$ 8) $\dfrac{\sqrt{x+1}+1}{x}$

D. Simplify the rational expression:

9) $\dfrac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

Solve: You may use your graphing calculator to check solutions.

10) $(x+3)^2 > 4$ 11) $\dfrac{x+5}{x-3} \le 0$ 12) $3x^3 - 14x^2 - 5x \le 0$ (Factor first)

13) $x < \dfrac{1}{x}$ 14) $\dfrac{x^2-9}{x+1} \ge 0$ 15) $\dfrac{1}{x-1} + \dfrac{4}{x-6} > 0$

16) $x^2 < 4$ 17) $|2x+1| < \dfrac{1}{4}$

F. Solve the system. Solve the system algebraically and then check the solution by graphing each function and using your calculator to find the points of intersection.

18) $x - y + 1 = 0$
$y - x^2 = -5$

19) $x^2 - 4x + 3 = y$
$-x^2 + 6x - 9 = y$

II. Graphing and Functions:

A. Linear graphs: *Write the equation of the line described below.*

20) Passes through the point (2, -1) and has slope $-\frac{1}{3}$.

21) Passes through the point (4, - 3) and is perpendicular to $3x + 2y = 4$.

22) Passes through (-1, - 2) and is parallel to $y = \frac{3}{5}x - 1$.

B. Functions: *Find the domain of the following.*

Note: domain restrictions - denominator $\neq 0$, argument of a log or ln > 0,
radicand of even index must be ≥ 0

23) $y = \frac{3}{x - 2}$ 24) $y = \log(x - 3)$

25) $y = \sqrt{2x - 3}$ 26) $y = \frac{\sqrt{x+1}}{x^2 - 1}$

27) Given f(x) below, sketch the graph over the domain [-3, 3].

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 1 & \text{if } -1 \leq x < 0 \\ x - 2 & \text{if } x < -1 \end{cases}$$

Find the composition /inverses as indicated below.

Let **f(x) = $x^2 + 3x - 2$** **g(x) = 4x - 3** **h(x) = ln$x$** **w(x) = $\sqrt{x - 4}$**

28) $g^{-1}(x)$ 29) $h^{-1}(x)$ 30) $w^{-1}(x)$, for $x \geq 4$ 31) f(g(x)) 32) h(g(f(1)))

33) Does $y = 3x^2 - 9$ have an inverse function? Explain your answer.

Let **f(x) = 2x, g(x) = -x, and h(x) = 4,** find

34) $(f \circ g)(x)$ 35) $(f \circ g \circ h)(x)$

36) Let $s(x) = \sqrt{4 - x}$ and $t(x) = x^2$, find the domain and range of $(s \circ t)(x)$.

C. Basic Shapes of Curves:

Sketch the graphs. You may use your graphing calculator to verify your graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

37) $y = \sqrt{x}$ 38) $y = \ln x$ 39) $y = \frac{1}{x}$ 40) $y = |x - 2|$

41) $y = \frac{1}{x - 2}$ 42) $y = \frac{x}{x^2 - 4}$ 43) $y = e^{-x}$

44) $f(x) = \begin{cases} \sqrt{25 - x^2} & \text{if } x < 0 \\ \frac{x^2 - 25}{x - 5} & \text{if } x \geq 0,\ x \neq 5 \\ 0 & \text{if } x = 5 \end{cases}$

III. LOGARITHMIC AND EXPONENTIAL FUNCTIONS

A. Simplify Expressions: *Non-calculator*

45) $\log_4\left(\frac{1}{16}\right)$ 46) $3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3\left(\frac{1}{27}\right)$ 47) $\log_9 27$

48) $\log_{125}\left(\frac{1}{5}\right)$ 49) $\log_w w^{45}$ 50) $\ln e$ 51) $\ln 1$ 52) $\ln e^2$

B. Solve equations: Non-calculator

53) $\log_6(x + 3) + \log_6(x + 4) = 1$ 54) $\log x^2 - \log 100 = \log 1$ 55) $3^{x+1} = 15$

IV TRIGONOMETRY

A. Unit Circle: Know the unit circle – radian and degree measure. Be prepared for a quiz.

56) State the domain, range and fundamental period for each function?

a) $y = \sin x$ b) $y = \cos x$ c) $y = \tan x$

B. Solve the Equations

57) $\cos^2 x = \cos x + 2, \quad 0 \le x \le 2\pi$ 58) $2\sin(2x) = \sqrt{3}, \quad 0 \le x \le 2\pi$

59) $\cos^2 x + \sin x + 1 = 0, \quad 0 \le x \le 2\pi$

C. Inverse Trig Functions: Note: $\sin^{-1} x$ = Arcsin x

60) Arcsin1 61) $\text{Arcsin}\left(-\frac{\sqrt{2}}{2}\right)$ 62) $\text{Arccos}\left(\frac{\sqrt{3}}{2}\right)$ 63) $\sin\left(\text{Arccos}\left(\frac{\sqrt{3}}{2}\right)\right)$

64. State domain and range for: Arcsin(x) , Arccos (x), Arctan (x)

D. Be able to do the following on your graphing calculator:

Be familiar with the CALC commands; value, root, minimum, maximum, intersect. You may need to zoom in on areas of your graph to find the information.
Answers should be accurate to 3 decimal places. Sketch graph.

65 -68. Given the following function $f(x) = 2x^4 - 11x^3 - x^2 + 30x$.

65. Find all roots.

66. Find all local maxima.

67. Find all local minima.

Note: Window x min: -10 x max: 10 scale 1
y min: - 100 y max: 60 scale 0

A local maximum or local minimum is a point on the graph where there is a highest or lowest point within an interval such as the vertex of a parabola

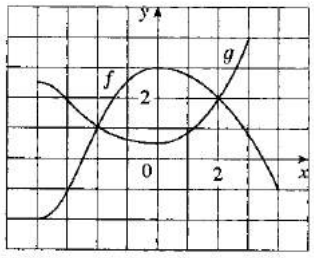
68. Find the following values: f (-1), f (2), f (0), f (.125)

69. Graph the following two functions and find their points of intersection using the intersect command on your calculator.

$y = x^3 + 5x^2 - 7x + 2$ and $y = .2x^2 + 10$ Window: x min : -10 x max: 10 scale 1
y min: - 10 y max: 50 scale 0

V. Functions and Models

70. The graphs of f and g are given.



(a) State the values of f(-4) and g(3).
(b) For what values of x if f(x)=g(x)?
(c) Estimate the solution of the equation f(x) = -1.
(d) On what interval is f decreasing?

(e) State the domain and range of f.
(f) State the domain and range of g.

71. If $f(x) = 3x^2 - x + 2$, find f(2), f(-2), f(a), f(-a), f(a+1), 2f(a), $f(a^2)$, $[f(a)]^2$, and f(a+h).

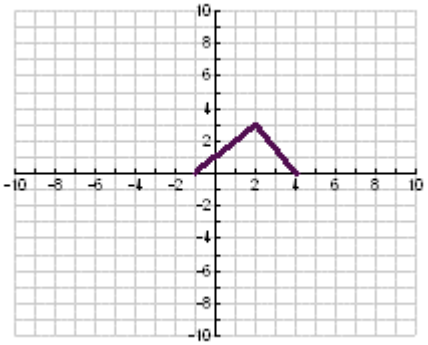
72. Find the domain of each function.

a) $f(x) = \dfrac{x}{3x - 1}$

b) $g(u) = \sqrt{u} + \sqrt{4 - u}$

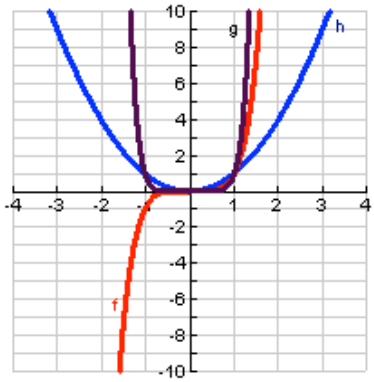
73. Find an expression for the bottom half of the parabola $x + (y-1)^2 = 0$.

74. Find an expression for the function whose graph is the given curve.



75. Match each equation with its graph. Explain your choices. (Don't use a computer or graphing calculator).

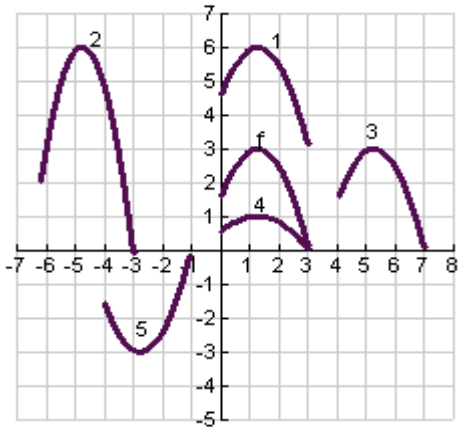
(a) $y = x^2$ (b) $y = x^5$ (c) $y = x^8$



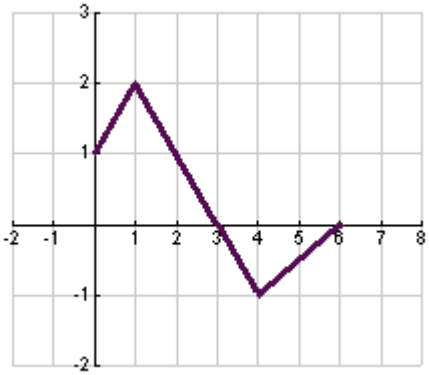
76. Suppose the graph of f is given. Write equations for the graphs that are obtained from the graph of f as follows.

(a) Shift 3 units upward. (b) Shift 3 units downward.
(c) Shift 3 units to the right. (d) Shift 3 units to the left.
(e) Reflect about the x-axis. (f) Reflect about the y-axis.
(g) Stretch vertically by a factor of 3. (h) Shrink vertically by a factor of 3.

77. The graph of y = f(x) is given. Match each equation with its graph and give reasons for your choices.
(a) y = f(x-4) (b) y = f(x)+3 (c) y = 1/3 f(x) (d) y = -f(x+4) (e) y = 2f(x+6)



78. The graph of f is given. Use it to graph the following functions.
(a) y = f(2x) (b)=f(½ x) (c) y = f(-x) (d) y = -f(-x)



79. Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.
$f(x) = \sin x, \qquad g(x) = 1 - \sqrt{x}$

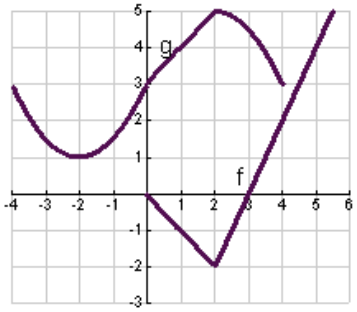
80. Express the function in the form $f \circ g$.

$F(x) = (x^2 + 1)^{10}$

81. Use the given graphs of f and g to evaluate each expression, or explain why it is undefined.

(a) f(g(2)) (b) g(f(0))

(c) $(f \circ g)(0)$ (d) $(g \circ f)(6)$ (e) $(g \circ g)(-2)$ (f) $(f \circ f)(4)$



82. Graph the ellipse $4x^2 + 2y^2 = 1$ by graphing the functions whose graphs are the upper and lower halves of the ellipse.

83. Use your calculator to find all solutions of the equation correct to three decimal places.

$x^3 - 9x^2 - 4 = 0$

84. Starting with the graph of $y = e^x$, write the equation of the graph that results from

(a) shifting 2 units downward

(b) shifting 2 units to the right

(c) reflecting about the x-axis

(d) reflecting about the y-axis

(e) reflecting about the x-axis and then about the y-axis

For # 85-87, find a formula for the inverse of the function.

85. $f(x) = \sqrt{10 - 3x}$

86. $f(x) = e^{x^3}$

87. y = ln(x+3)

For # 88-89, find the exact value of each expression (non-calculator).

88. (a) $\log_2 64$ (b) $\log_6 \frac{1}{36}$

89. (a) $\log_{10} 1.25 + \log_{10} 80$ (b) $\log_5 10 + \log_5 20 - 3\log_5 2$

90. Express the given quantity as a single logarithm.
2 ln 4 – ln 2

Answers: (Remember – you must show all of your work!)

1. $4x^{11/3}y^{4/3}z$ 2. (3x + 1)(3x - y) 3. $(2x - 1)(4x^2 + 2x + 1)(2x + 1)(4x^2 - 2x + 1)$

4. $7(3x^2 + 4)(2x^2 - 1)$ 5. $x^{1/2}(3x - 4)(5x + 6)$

6. $x^{-3}(x - 2)(x - 1)$ 7. $1 + \sqrt{x - 2}$ 8. $\frac{1}{\sqrt{x+1}-1}$ 9. $\frac{x^2 - x + 1}{(x + 1)^2}$

10. x> -1 or x < -5 11. -5 ≤ x < 3 12. $x \le -\frac{1}{3}$ or 0 ≤ x ≤ 5 13. 0 < x < 1 or x < -1

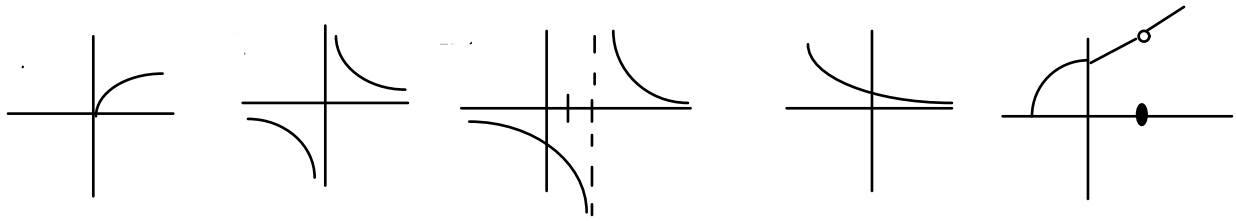
14. [-3, -1) U [3, ∞) 15. x > 6 or 1 < x < 2

16. –2 < x < 2 17. $-\frac{5}{8} < x < -\frac{3}{8}$ 18. (3, 4), (-2, -1) 19. (2, -1) , (3, 0)

20. $y = -\frac{1}{3}x - \frac{1}{3}$ 21. $y = \frac{2}{3}x - \frac{17}{3}$ 22. $y = \frac{3}{5}x - \frac{7}{5}$ 23. D: x ≠ 2 24. $D: x > 3$

25. D: $x \ge 3/2$ 26. x >-1 and x ≠ 1 27. R: - 5 ≤ y < - 3 or 0 ≤ y ≤ 3 28. $g^{-1}(x) = \frac{x+3}{4}$

29. $h^{-1}(x) = e^x$ 30. $y = x^2 + 4$ $x \ge 0$, 31. $f(g(x)) = 16x^2 - 12x - 2$ 32. ln 5
33. no, explain: this function Is not one-to-one (pass the horizontal line test)
34. –2x 35. - 8 36. D: $-2 \le x \le 2$ R: $0 \le y \le 2$
37. 39. 41. 43. 44.



45. - 2 46. –1 47. 3/2 48. – 1/3 49. 45 50. 1 51. 0 52. 2 53. - 1

54. x = 10, -10 55. $\frac{\log 15}{\log 3}$ - 1 56. a) D: all reals, R: $-1 \le x \le 1$, 2 л

b) D: all reals, R: $-1 \le x \le 1$, 2 л c) D: $x \neq \pi/2$ R: all reals , π

57. π 58. $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$ 59. $\frac{3\pi}{2}$ 60. $\frac{\pi}{2}$

61. $-\frac{\pi}{4}$ 62. $\frac{\pi}{6}$ 63. $\frac{1}{2}$ 64. Arcsin(x) D: [-1, 1] Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$,

Arc cos (x) D: [-1, 1]. R: [0, π] Arctan (x) D: all reals R: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

65. - 1.5, 0 , 2 , 5 66. rel max. (1.07, 20.1) 67. rel. min (- .89, - 18. 48), (3.94, - 88)

68. f(-1) = - 18 f(2)=0 f(0)=0 f(0.125)=3.7133 69. 3 points of intersection - one is (-5.77, 16.66)

69. Do not use trace to find points. Use CALC commands.

70. (a) -2,4 (b) -2 and 2 (c) x= -3 and 4 (d) [0,4] (e) d: [-4,4] r:[-2,3] (f) d: [-4,3] r:[0.5,4]
71. f(2)=12, f(-2) = 16, f(a) = 2a^2 -a +2,
f(-a)=3a^2+a+2, f(a+1)=3a^2+5a+4, 2f(a)=6a^2-2a+4, f(2a)=12a^2-2a+2, f(a^2)=3a^4 -a^2 +2,
[f(a)]^2=9a^4 -6a^3+13a^2-4a+4, f(a+h)=3a^2 +6ah+3h^2 -a-h+2
72. (a) (-∞, 1/3) U (1/3,∞) (b) [0,4] 73. $f(x) = 1 - \sqrt{-x}$ (domain: $x \le 0$)
74. $f(x) = \begin{cases} x+1 & if \ -1 \le x \le 2 \\ -\frac{3}{2}x + 6 & if \quad 2 < x \le 4 \end{cases}$
75. (a) matches with h (b) matches with f and (c) matches with g.
76. (a) y = f(x)+3 (b) y = f(x) -3 (c) y = f(x-3) (d) y = f(x+3) (e) y = -f(x)
(f) y = f(-x) (g) y = 3f(x) (h) y = 1/3 f(x) 113. (a) graph 3 (b) graph 1 (c)graph 4
(d) graph 5 (e) graph 2 77. (a) shrink horizontally by a factor of 2 (b) stretch horizontally by a factor of 2 (c) reflect the graph of f about the y-axis (d) reflect the graph of f about the y-axis, then about the x-axis

79. (f+g)(x) =x^3+5x^2-1 d: all real numbers (f-g)(x)= x^3-x^2+1 d: all reals
(fg)(x)=3x^5+6x^4-x^3-2s^2 d: all reals (f/g)(x)=(x^3 +2x^2)/(3x^2 -1) d: x cannot
Equal $\pm\frac{1}{\sqrt{3}}$ 79. $(f \circ g)(x) = \sin(1 - \sqrt{x})$ $d: [0, \infty)$ $(g \circ f)(x) = 1 - \sqrt{\sin x}$ $d: [0, \pi], [2\pi, 3\pi]$ *etc*

$(f \circ f)(x) = \sin(\sin x)$ $d: (-\infty, \infty)$ $(g \circ g)(x) = 1 - \sqrt{1 - \sqrt{x}}$ $d: [0,1]$ 80. g(x)=x^2 +1 and F(x)=x^10 81. (a) 4 (b) 3 (c)0 (d) not defined (e) 4 (f) -2 82. graph y = +sqr((1-4x^2)/2) and -sqr((1-4x^2)/2) 83. about 9.05 84. (a) y = e^x -2 (b) y = e^(x-2) (c) y = -e^x

(d) y = e^(-x) (e) y = - e^(-x) 85. $f^{-1}(x) = -\frac{1}{3}x^2 + \frac{10}{3}$ d: [0,∞) 86. $f^{-1}(x) = \sqrt[3]{\ln x}$ 87. $f^{-1}(x) = e^x - 3$

88 (a) 6 (b) -2 89. (a) 2 (b) 2 90. ln8